

LOW COST PRODUCTION PHASE NOISE MEASUREMENTS ON MICROWAVE AND MILLIMETRE WAVE FREQUENCY SOURCES

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1. Introduction

Production test on high frequency components and sub-systems is most efficiently performed using dedicated test benches which once set up and calibrated remain undisturbed for the duration of the contract. Although ideal, this approach can incur high capital equipment costs if several jobs are running in parallel through production.

One of the most expensive pieces of test gear is often the Phase Noise Measurement System, good quality commercial instrumentation for this purpose being designed for high performance and the ability to cope with a variety of different frequency ranges and measurement situations. In a production test environment, however, the measurement requirement for a particular item is usually quite specific, and it is often possible to take advantage of this fact to substantially reduce capital equipment cost. The object of this paper is to provide an example by showing how accurate phase noise measurements on microwave and millimetre wave frequency sources may be performed at very low cost by means of the Two-Source I.F. Discriminator Method. Following a review of phase noise measurement techniques, design of the set-up is discussed and its use illustrated with reference to measurements on three different types of low noise microwave source.

2. Background

Phase Noise Measurements on low noise frequency sources are generally performed by either the Two-Source Phase Detector Method or by the Single Source Frequency Discriminator Method.

2.1 Two-Source Phase Detector Method (Sources at Same Frequency)

This method exhibits the lowest measurement noise floor and is the basis of most commercial instrumentation. The simplest set-up is shown in Figure 1. Here, two sources, both at the same frequency, are held locked in phase by a low bandwidth phase locked loop, hence the measurement is only valid outwith the locking loop bandwidth. This restriction may be overcome by measuring the loop transfer function and correcting the close to carrier data accordingly. This is done automatically in commercial instrumentation.

The spectrum analyser is set to measure the power spectral density of the voltage fluctuations at the LNA output, resulting in a direct reading of phase noise after appropriate scaling for the PSD constant and LNA gain. Some FFT analysers may only display the spectral density of voltage fluctuations, in which case the squaring to yield power must be done in software. Because the phase noise of a typical source may change by more than 100 dB over the measurement offset frequency range of interest, it is common to gather the raw data in decades of frequency in order to reduce the spectrum analysis dynamic range and frequency resolution requirements.

To avoid the need for one of the sources to have phase noise substantially lower than the source under test, it is also common practice to measure pairs of nominally identical sources and subtract 3 dB from the result on the assumption of similar noise performance. This is usually an acceptable procedure for production test measurements, since in the worst case neither source will have phase noise higher than the measured result, and an appreciable margin will usually have been allowed between expected performance and the pass/fail limit.

The Two-Source Phase Detector Method although generally good is dependent on the sources being sufficiently stable to hold lock satisfactorily, and on being sufficiently well isolated to prevent injection locking. It also depends on accurate calibration of the PSD constant, normally performed by

measuring the amplitude of the difference frequency at the PSD output with the locking loop open. For a sinusoidal waveform the slope at zero volts is equal to the peak amplitude, hence the phase detector constant may be easily obtained. It is common practice, however, to drive both ports of the PSD as hard as possible in order to minimise the noise floor, in which case the PSD response will be non-sinusoidal. In this case calibration must be performed against a known phase change.

2.2 Two-Source Phase Detector Method (Sources Separated in Frequency)

Figure 2 shows the set-up required if it is not possible to electronically tune one of the sources, or if the tuning bandwidth is insufficient. Here two sources differing in frequency are mixed to an I.F. and the phase noise measurement performed using a low noise signal generator with DCFM as the tunable source. The performance of this instrument sets a limit on the measurement noise floor. Sometimes it is beneficial to run the generator at a higher frequency and employ low noise external pre-scalars to divide down to the I.F..

2.3 Single-Source Frequency Discriminator Method (Delay Line Discriminator)

This method, shown in Figure 3, uses a delay line frequency discriminator to convert source frequency variations to phase variations which are detected by the PSD. The PSD output voltage fluctuations are then directly proportional to the source frequency fluctuations. The spectrum analyser is set to measure the spectral density of these voltage fluctuations, i.e. the F.M. noise. Software is then employed to make the conversion to phase noise by the formula shown. Note that a 20 dB / decade linear fall-off in source phase noise with increasing offset from carrier results in a flat FM noise result, thereby reducing the analyser dynamic range requirement relative to the direct measurement of phase noise.

The delay line discriminator method is attractive in that it can tolerate drift in source frequency over the duration of the measurement. As the delay line length is increased the noise floor improves at the expense of more limited measurement bandwidth.

2.4 Single Source Frequency Discriminator Method (Resonator Discriminators)

Delay line loss increases with frequency, necessitating the use of mechanically tunable resonators to produce the frequency to phase conversion instead of delay lines in microwave discriminators. Transmission resonators are the most straightforward to use and give optimum discriminator sensitivity when the input/output port couplings are equal and set for 6 dB transmission loss at resonance.

Discriminator sensitivity increases with input power level. This is limited, however, to 6 dB above the PSD maximum in the case of the transmission resonator. Using a reflection resonator allows a substantial increase in discriminator input power and hence sensitivity. Here, a single-port resonator, usually a high Q cavity, is matched at resonance and operates in conjunction with a 3-port circulator to form a notch filter. With the source power initially attenuated to a safe level, the cavity is tuned to place the signal in the centre of the notch and the source power then increased. The PSD now functions as a bi-polar A.M. detector, detecting the change in amplitude with frequency on each side of the notch. Very high sensitivity is possible, resulting in a low noise floor, but great care must be taken to keep the signal in the centre of the notch to avoid destroying the PSD.

3. **The Two-Source I.F. Discriminator Method**

3.1 General Description

This method, which forms the main subject of this paper, overcomes the problem of excessive delay line loss at microwave frequencies in the normal single-source discriminator method described in paragraph 2.3 above. The technique is by no means new, but seems to be relatively unknown and unused owing to the predominance of the two-source phase detector method in commercial instrumentation and in the literature. The basic set-up is shown in Figure 4.

As in the two-source phase detector variant shown in Figure 2, the method depends on having two sources available for test, separated in frequency by a suitable amount. The two sources are mixed to

form an I.F., and the phase noise of this signal measured using a delay line discriminator operating at a much lower frequency than the original sources. A 100 MHz Ultra-Low Noise Crystal Oscillator is included for system noise floor measurement by injecting into the IF amplifier input port in place of the normal signal from the mixer.

The Two-Source I.F. Discriminator Method lends itself well to a low cost solution for production phase noise measurements on microwave and millimetre wave frequency sources. Apart from attenuators to set suitable signal levels, the only microwave component required is a simple mixer. Unlike the corresponding phase detector method with two sources differing in frequency, there is no loop to maintain in lock and to have to calibrate for close in measurements. Unlike the single source discriminator method, delay line loss is not a problem at the relatively low I.F., and the cable for the line is inexpensive. In addition, the noise floor is readily verified and can be shown on every source phase noise plot.

On the negative side, as for the single source discriminator method, the noise floor rises at 20 dB / decade towards carrier, and at a higher slope closer-in, as flicker noise takes effect. The bandwidth of the discriminator also becomes narrower as the line length is increased to obtain a lower noise floor. Despite these two factors, however, it will be shown that the technique is still a very good one for production measurements on microwave and millimetre wave sources, and can be implemented at very low cost and improved by the use of modern cross-correlation techniques in the software.

3.2 Practical System

Figure 5 shows a block diagram of the hardware in use at Spectral Line Systems Ltd for production phase noise measurements on microwave sources operating in the frequency range 10 to 15 GHz.

The two sources to be measured are attenuated to give 0 dBm and + 10 dBm at the mixer input ports, resulting in around -8 dBm at the mixer output. This is amplified to a power level of + 30 dBm in a bipolar transistor limiting amplifier with noise figure 5.5 dB and flat frequency response from 50 to 250 MHz. A harmonic filter follows the amplifier to ensure a sinusoidal waveform enters the discriminator bridge. This is important, since a distorted input waveform can give rise to irregularities in the PSD response.

Delay line lengths ranging from 5 m up to 100 m are normally employed in the system, depending on the noise floor and offset frequency range over which the measurement is required. Occasionally 200 m is used to obtain a lower noise floor, although this requires an increase in discriminator input power. We have found RG 213 A/U to be a suitable cable for the line, providing a good compromise between price and performance. This is a 10.3 mm O.D. 50 Ohm cable of solid polythene dielectric and braided copper screen construction. The velocity factor is 0.66, the loss for a 100 m reel measuring 4.2 dB at 50 MHz, 6.1 dB at 100 MHz and 9.0 dB at 200 MHz. Note that there is no point in using a more expensive semi-rigid cable in this frequency range. The corresponding loss figures for UT 141 cable, for example, are 7.7 dB at 50 MHz, 11 dB at 100 MHz and 15.7 dB at 200 MHz.

The discriminator bridge includes a 5-bit digitally controlled line stretcher with a phase increment of one degree at 100 MHz. This is used (in conjunction with an external cable if necessary) to set the PSD output voltage to near zero before performing a measurement. The PSD is a Mini-Circuits TFM4-H driven with both ports at + 17 dBm, followed by a 20 MHz low pass filter and baseband amplifier. The amplifier consists of an LT 1028 low noise non-inverting op. amp. of x 5 voltage gain for measurements out to 100 KHz and a 30 dB gain AC coupled MMIC amplifier for 0.1 to 10 MHz.

Before measuring a production batch of sources the system noise floor is measured by switching the I.F. amplifier input to the signal from a 100 MHz Ultra-low Noise Voltage Controlled Crystal Oscillator of our own manufacture, attenuated to the same power level as the mixer output signal. The oscillator tuning (+/- 1 KHz) may be used to calibrate the discriminator when used with long delay lines giving a high sensitivity. Otherwise an external signal generator is employed.

Spectrum analysis is performed using a Stanford Research Systems SR 760 FFT Spectrum Analyser for measurements out to 100 KHz, and an analogue instrument such as the Agilent HP 8563E at

greater offset frequency ranges. We normally allow an overlap of 1 decade in frequency when using both analysers, the degree of matching between the two sets of results providing some confidence in the measurement. When using the FFT a simple single pole 250 KHz RC low pass filter is inserted between the baseband amplifier output and the analyser input. This is necessary because the instrument uses active anti-aliasing filters, which are not effective above the bandwidth of the op. amps. employed for their realisation. We have found that sources with phase noise which rises at offsets greater than 100 KHz give erroneous results if this filter is not included.

The hardware is constructed using parts drawn from our “Curikela” range of RF Circuit Kits developed for use in test gear and for university and college projects. These consist of a modularised system allowing RF components and sub-systems to be built at low cost to a good standard with no pcb manufacture or machining being involved. Using this approach, we have found it most cost effective to build a specific fixed set up for each production measurement job, rather than attempt to produce a more complicated system aimed at more universal application.

3.3 System Design and Performance

Appendix 1 lists a number of basic relationships for delay line discriminator design. Note that the discriminator magnitude response to an input signal with sinusoidal FM follows a $\sin x/x$ function as the modulating frequency is increased. Table 1 shows the response for the various line lengths commonly employed in the system. Normally the line length would be restricted to one giving less than 1 dB of noise suppression at the highest offset frequency.

Having chosen the line length, the measurement noise floor must be checked to ensure that it is adequate for the task envisaged. This may be done during the system design stage by calculation based on a knowledge of the PSD added phase noise.

To measure the PSD noise floor the delay line is replaced with an attenuator equal in value to the line loss, the bridge phased for quadrature, and the discriminator excited with the signal from the amplified low noise crystal oscillator. Note that added phase noise from the power amplifier is common to both inputs to the PSD and therefore does not affect the result for the PSD itself. A measurement of the PSD noise floor for the Mini-Circuits TFM 4-H device used in the system described above when driven at + 17 dBm on both ports is shown in Figure 6.

The corresponding discriminator phase noise measurement floor may be found by:

- (a) Increasing the PSD result by the factor $- 20 \text{ Log}_{10} (2 \cdot \pi \cdot f_m \cdot T_d)$,

Where f_m = Offset from carrier (Hz)
 T_d = Delay Time (sec.)

This expression is derived in Appendix 1.

- (b) Modifying the curve obtained above by imposing a lower limit set by the added phase noise of the power amplifier between the mixer and discriminator. For a mixer output signal level of - 8 dBm and an amplifier noise figure of 5 dB, as shown in the system block diagram of Fig. 5, this limit is - 164 dBc / Hz, since thermal phase noise is - 177 dBm / Hz.

Figure 7 shows the discriminator noise floor for the case of a 100 m delay line of RG 213 A/U, as measured directly by injecting the low noise crystal oscillator signal at - 8 dBm into the I.F. amplifier input port. Values for the discriminator floor calculated from the measured PSD floor by the procedure described above are indicated by dots on the plot, and agree with the directly measured result, assuming the curve eventually flattens out at - 164 dBc / Hz.

Figure 8 shows the noise floor calculated in the above manner for a number of different delay line lengths, based on the same PSD noise as measured in Figure 6. This provides a guide to the performance which can be expected when considering a production measurement by the Two-Source I.F. Discriminator Method.

4 Examples of Production Phase Noise Measurements on Microwave Sources

In order to demonstrate the effectiveness of the Two-Source IF Discriminator Method some examples of its application to Spectral Line Systems Ltd production sources are now described.

4.1 Low Noise Crystal Multiplier Sources

These sources are based on an Ultra-Low Noise Crystal Oscillator similar to that employed in the measurement system described earlier followed by frequency multiplication to microwave. A typical scheme for a 10 GHz source, for example, would be a 125 MHz oscillator multiplied by 8 in successive diode doublers and then by 10 in a step recovery diode multiplier. A good source of this type will have phase noise $20 \text{ Log}_{10} 80 = 38 \text{ dB}$ above the oscillator noise, degraded by around 3 dB to 5 dB over the 10 to 100 KHz region, owing to the added phase noise of the multiplier. Typical figures are tabulated below and compared with the measurement discriminator noise floor:

Offset, Hz	Phase Noise, dBc / Hz				
	Crystal Osc. At 125 MHz	Mult. O/P at 10.0 GHz	2-Source Result at 10.0 GHz	Disc. Floor 200 m Line (From Figure 7)	Margin dB
100	- 125	- 87	- 84	- 94	10
1 K	- 155	- 117	- 114	- 124	10
10 K	- 168	- 127	- 124	- 152	28
100 K	- 170	- 130	- 127	- 164	37

Note that a discriminator line length of 200 m is necessary to provide a margin of 10 dB in this case. Table 1 shows that a 200 m line still gives adequate bandwidth at 100 KHz.

4.2 Ultra-Low Phase Noise Discriminator Stabilised Sources

These sources are based on a resonator discriminator noise control loop which reduces the phase noise of a low noise voltage tuned microwave source to an even lower level. Typical phase noise at 15.0 GHz is as shown below, and again compared with the measurement discriminator noise floor:

Offset, Hz	Phase Noise, dBc / Hz at 15.0 GHz			
	Ultra-Low Noise Source	2-Source Result	Disc. Floor 100 m Line (From Fig. 7)	Margin dB
1 K	- 110	- 107	- 119	12
10 K	- 140	- 137	- 146	9
100 K	- 147	- 144	- 164	20

Production systems operate on a number of different frequency channels spaced in frequency such that it is possible to measure them in pairs with suitable frequency differences for the measurement discriminator, which is based on a 100 m reel of RG 213 A/U cable, as described previously. The table above shows that this length of line is just sufficient to perform the measurement.

4.3 Delay Line Discriminator Stabilised Microwave Sources

These sources consist of a delay line discriminator stabilised VCO at 1.0 GHz followed by a step recovery diode frequency multiplier to 15 GHz. The driving force behind this design is a requirement for the source to be tunable by at least $\pm 20 \text{ MHz}$, and to have phase noise in the 1.0 to 10.0 MHz offset region lower than can be obtained from the best crystal multiplier. Production sources are measured in pairs tuned 40 MHz apart, yielding an I.F. of 80 MHz for the Two Source I.F. Discriminator measurements. A 30 m delay line is employed for measurements out to 100 KHz offset using a Stanford Research Systems SR 760 FFT, and a 5 m line for measurements from 100 KHz to 10 MHz, using an HP 8563E spectrum analyser, typical results being as shown below:

Offset, Hz	Phase Noise, dBc / Hz				
	Source at 1.0 GHz	Mult. O/P at 15.0 GHz	2-Source Result at 15.0 GHz	Disc. Floor 30 m Line	Margin dB
1 K	- 96	- 72	- 69	- 109	40
10 K	- 120	- 96	- 93	- 136	43
100 K	- 141	- 117	- 114	- 160	46
				5 m Line	
100 K	- 141	- 118	- 115	- 145	30
1 M	- 160	- 136	- 133	- 164	31
10 M	- 162	- 138	- 135	- 164	29

In early models of this source the 15.0 GHz signal was further multiplied up to 90 GHz in a bought-in x 6 frequency multiplier. At that time phase noise measurements were not performed on the W-Band signal, as we thought that we were not equipped for this measurement. In hindsight, with the Two-Source I.F. Discriminator Method, all that would have been required in the way of millimetre wave hardware would have been a simple mixer.

Note that it is always possible to measure a delay line stabilised source with a delay line discriminator, since the line used in the source can never be made as long as the measurement line, owing to excessive phase shift within the stabilisation loop.

5. Conclusion

This paper has shown that the phase noise of microwave sources may be measured at low cost by the Two-Source I.F. Discriminator Method, the measurement becoming easier as the source frequency increases. In the interests of both cost and reliability, a dedicated test set should be built for the particular job in hand, otherwise there is a danger of defeating the spirit of the exercise by attempting to effectively build an instrument.

Regarding future improvements to measurement noise floor, cross-correlation techniques may be applied to this method of phase noise measurement as effectively as to the Two-Source Phase Detector Method. Again in the interests of retaining simplicity and keeping costs low, however, it is probably better to settle for a cross-correlation scheme involving duplication of the discriminator, rather than the more complex arrangement of measuring using three sources. Hence the sources under test would continue to be measured in pairs and the discriminator noise floor reduced by cross-correlation.

The question of spectrum analysis for use with the measurement system is perhaps one which should also be addressed in this conclusion. At present we use the Stanford Research Systems SR760 single channel FFT for measurements up to 100 KHz and an HP 8563E for the 1 to 10 MHz range. It may be thought that the ideal instrument would be a two-channel FFT with good dynamic range, analysis up to at least 10 MHz, and the ability to perform cross-correlation measurements. Such an instrument, if available, would no doubt be expensive.

Given that two delay line lengths are necessary to cover the full analysis range, however, it is simpler and more cost effective to have two fixed hardware set ups, each of which can remain undisturbed, operating with its own spectrum analyser. Moderately priced instruments covering e.g. 9 KHz to 3.0 GHz are quite adequate for the upper range. By keeping the analyser price low it is possible to have spare instruments, providing cover for calibration periods. These instruments may also be used for general RF use on other production jobs when not in use for phase noise measurement.

When considering improving the noise floor by cross-correlation, this is only really worth doing on the lower offset frequency range, where it is possible to obtain a 100 KHz two-channel FFT as a stand-alone instrument, since the discriminator noise floor is usually much more than adequate at higher offset frequencies.

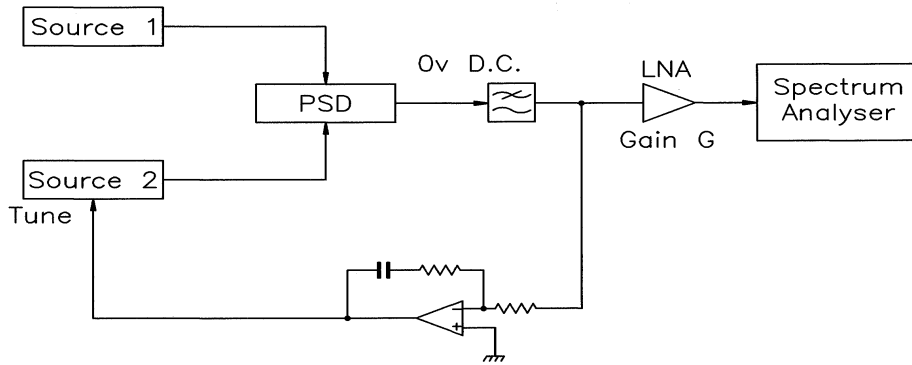


Fig. 1. Two-Source Phase Detector Method for Sources at Same Frequency.

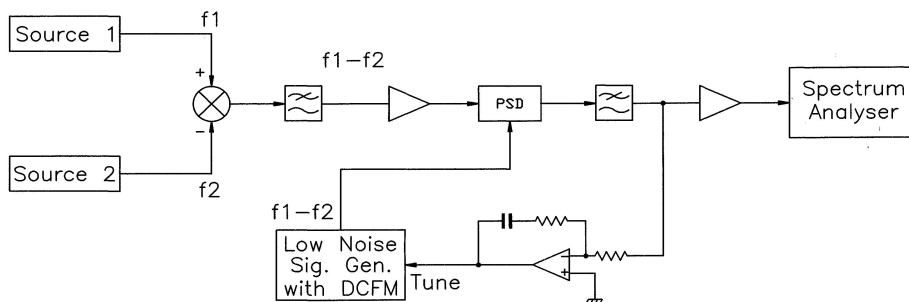


Fig. 2. Two-Source Phase Detector Method for Sources at Different Frequencies.

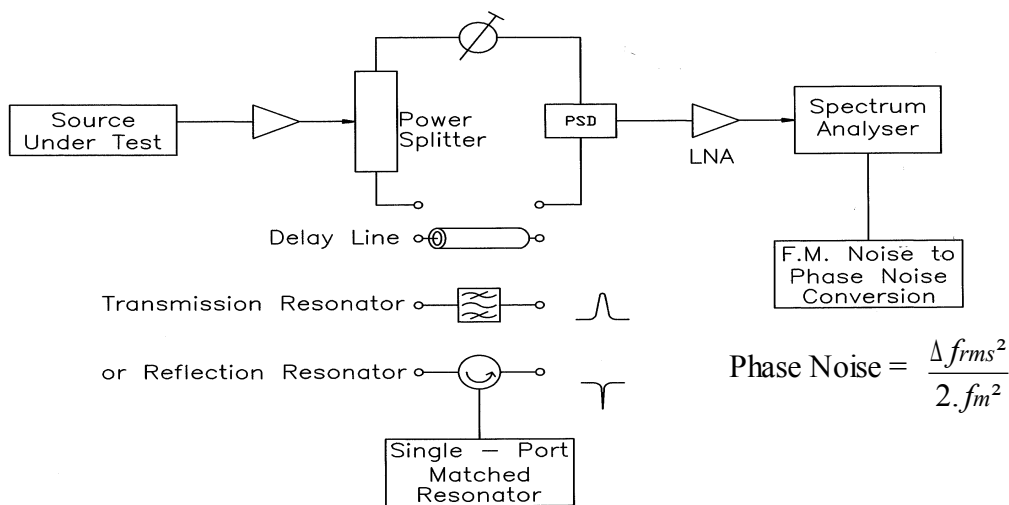


Fig. 3. Single-Source Frequency Discriminator Method.

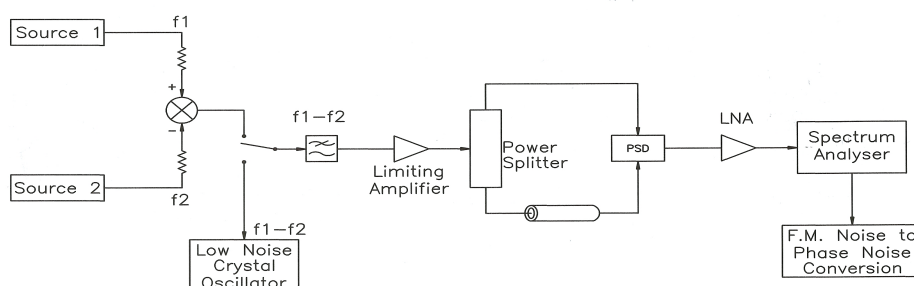


Fig. 4. Two-Source Frequency Discriminator Method.

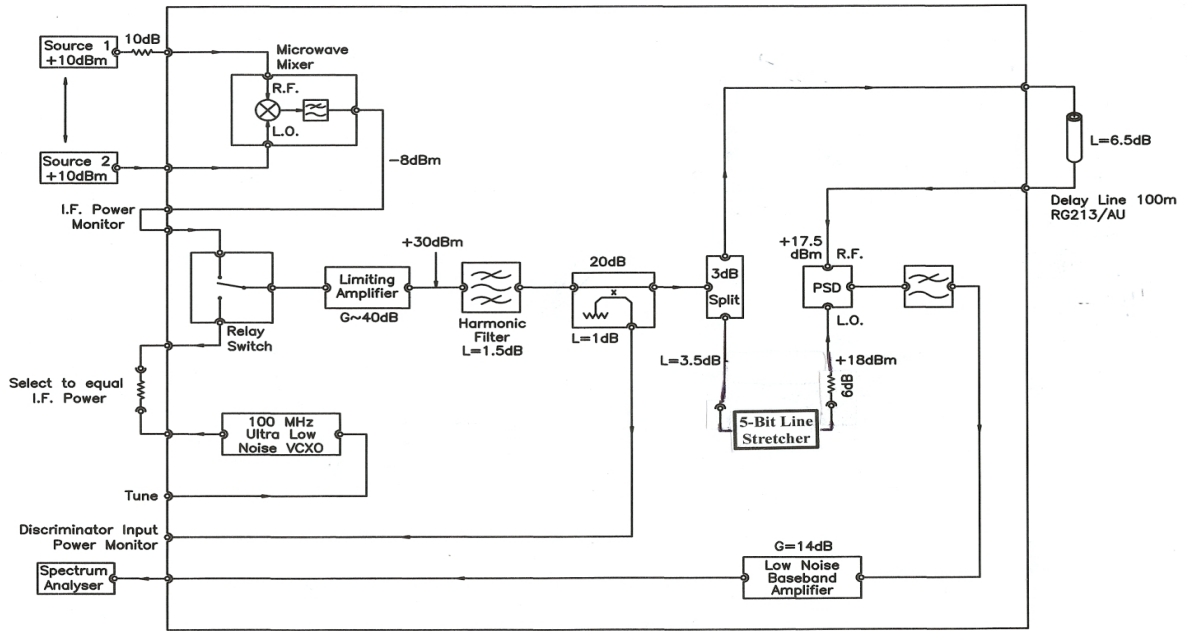


Figure 5. Test Set for 2-Source I.F. Discriminator Phase Noise Measurement

Figure 6.
PSD Noise Floor at 100 MHz
(Mini-Circuits TFM-4H PSD)

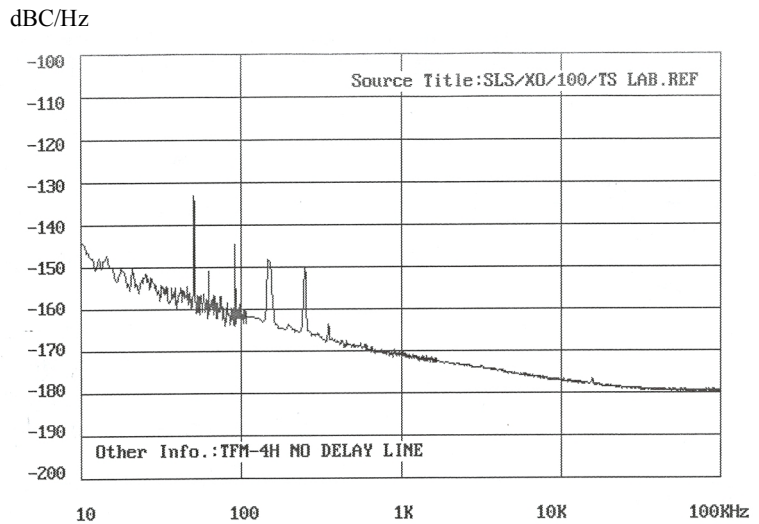
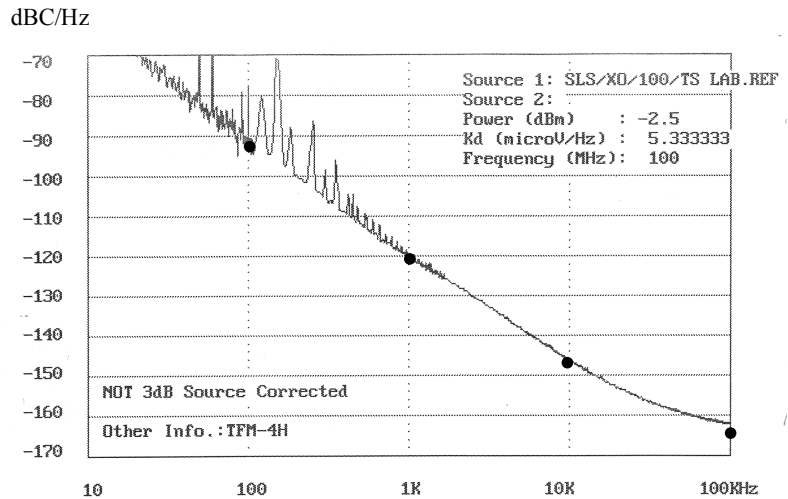


Figure 7.
Discriminator Noise Floor
at 100 MHz
(For System Shown in Fig. 5)

- Shows PSD Noise Floor of Fig. 6 raised by $-20\text{Log}_{10}(2\pi f m T d)$, except at 100 KHz where the dot is placed at the ultimate limit of -164 dBC/Hz, imposed by the I.F. Amp Noise Floor.



Length (m)	200	100	30	10	5	3
Delay $T_d, \mu\text{s}$	1.01	0.505	0.152	0.051	0.025	0.015
Freq. Hz	dB	dB				
100K	-0.15	-0.03				
200	-0.59	-0.15	dB			
400	-2.47	-0.59	-0.05			
600	-6.07	-1.35	-0.12	dB		
800	-12.98	-2.47	-0.21	-0.02	dB	
1M		-4.00	-0.34	-0.04	-0.01	dB
2M			-1.38	-0.15	-0.04	-0.01
4M			-6.21	-0.59	-0.15	-0.05
6M				-1.35	-0.33	-0.12
8M				-2.49	-0.59	-0.21
10M				-3.52	-0.92	-0.32
20M						-1.33
1st.Null, MHz	0.99	1.98	6.58	19.61	40.00	66.67

Table 1
Magnitude Response vs. Line Length with Velocity Factor 0.66

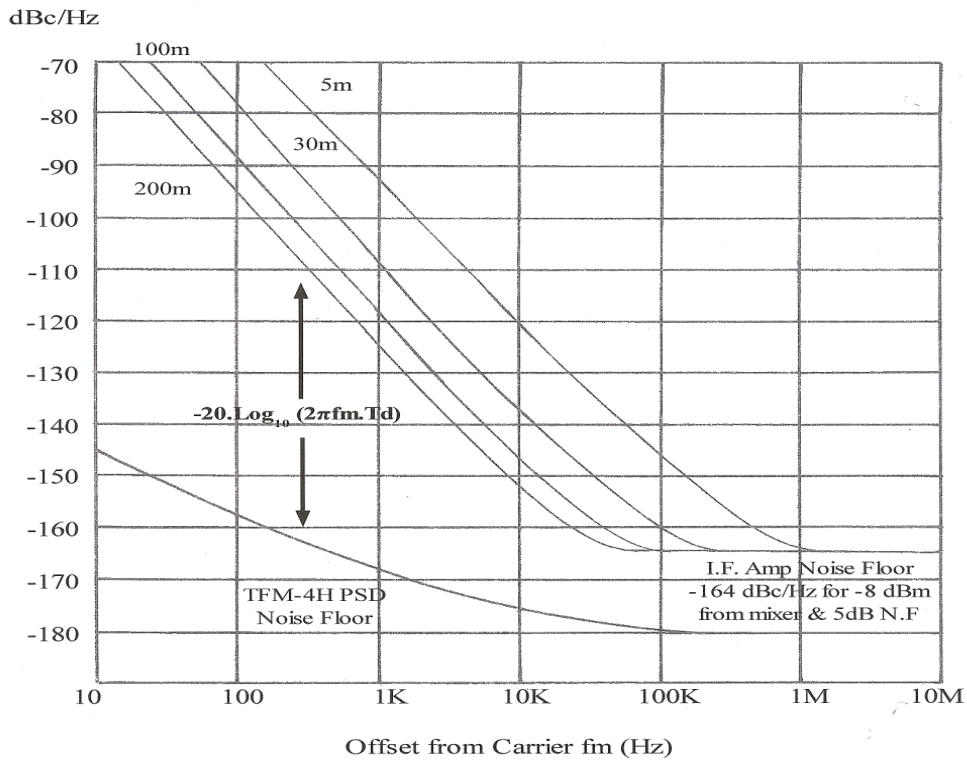


Figure 8. Discriminator Noise Floor for Various Lengths of Delay Line, Based on Measured PSD Residual Noise

T_d = Delay in Line (Sec.) Curves shown are for a Line with velocity factor 0.66

Appendix 1 - Delay Line Discriminator Design

Basic Relationships:

$$\text{Delay Time } T_d = \frac{\ell}{v} \text{ sec}$$

$$\text{Discriminator Constant } K_d = K_\phi \cdot 2\pi \cdot T_d \text{ v/Hz}$$

$$\text{Phase Shift Along Line } \phi = \frac{2\pi \cdot \ell}{\lambda_c} = 2\pi \cdot f_c \cdot T_d \text{ rad}$$

$$\text{Nulls Occur at: } \phi = (2n+1) \cdot \frac{\pi}{2} \text{ rad, } n=0,1,2,3\dots \quad \text{i.e. at: } f_c = \left(\frac{2n+1}{4} \right) \cdot \frac{1}{T_d} \text{ Hz}$$

where:

ℓ = Line Length (m)	f_c = Input Signal Freq. (Hz)
v = Velocity in Line (m/s)	λ_c = Input Signal Wavelength (m)
K_ϕ = Phase Detector Constant (v /rad)	

Response to Input Signal with Sinusoidal FM:

$$\text{Input Signal: } v_i(t) = v \sin(\omega_c t + m \cdot \sin \omega_m t)$$

$$\text{Output Signal Magnitude: } K_d \cdot \Delta f \cdot \frac{\sin(\pi \cdot f_m \cdot T_d)}{\pi \cdot f_m \cdot T_d} \quad \text{Nulls Occur at } f_m = \frac{n}{T_d}$$

$$\text{Output Signal Phase: } -\pi \cdot f_m \cdot T_d \quad \text{Phase at 1st Null} = \pi$$

Where:

ω_c = Carrier Ang. Frequency = $2\pi \cdot f_c$	m = Modulation Index = $\frac{\Delta f}{f_m}$
ω_m = Modulation Ang. Frequency = $2\pi \cdot f_m$	Δf = Peak Freq. Deviation

Noise Floor:

$$\text{Let PSD Residual Phase Noise} = N_{\text{PSD}} = \frac{\Delta \phi_{\text{rms}}^2}{2}$$

$$\text{This corresponds to a PSD output voltage of: } \Delta v_{\text{rms}}^2 = \frac{K_d^2 \cdot \Delta \phi_{\text{rms}}^2}{2} = 2 \cdot K_\phi^2 \cdot N_{\text{PSD}}$$

$$\text{Giving a Discriminator Phase Noise Floor of: } \frac{\Delta f_{\text{rms}}^2}{2 \cdot f_m^2} = \frac{\Delta v_{\text{rms}}^2}{2 \cdot K_d^2 \cdot f_m^2} = \frac{2 \cdot K_\phi^2 \cdot N_{\text{PSD}}}{2 \cdot K_d^2 \cdot f_m^2}$$

$$\text{Now } K_d = K_\phi \cdot 2\pi \cdot T_d, \quad \text{Hence Discriminator Floor is: } \frac{N_{\text{PSD}}}{(2\pi \cdot T_d \cdot f_m)^2}$$

i.e. To obtain the Discriminator Floor, add $|20 \log_{10}(2\pi \cdot T_d \cdot f_m)|$ dB to the PSD floor, subject to a lower limit imposed by the I.F. Amplifier noise figure and input power level.

Conversion From F.M. Noise to Phase Noise:

$$\text{Phase Noise} = 10 \log_{10} \left(\frac{\Delta f_{\text{rms}}^2}{2 \cdot f_m^2} \right)$$